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Doctoral thesis

H. W. Hoogstraten: **On Non-linear Dispersive Water Waves,***
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Author's Summary

In this thesis the propagation of three kinds of non-linear dispersive water waves is studied, viz. cnoidal waves, Boussinesq waves and Stokes waves.

The Introduction starts with a general survey of dispersive wave phenomena, followed by a summary of linear dispersive wave theory and Whitham's theory of non-linear conservative dispersive waves. Then a general asymptotic representation of slowly varying wavetrains is given. For such wavetrains quantities like wavenumber, frequency, amplitude, mean waveheight, etc. alter only by a very small fraction (of order of magnitude $1/K$, where K is a large number) of themselves during one period or within one wavelength. The asymptotic representation is in the form of a perturbation series in descending powers of the large parameter K.

Chapter I deals with asymptotic solutions of the Korteweg-de Vries equation for cnoidal waves in the form of a slowly varying wavetrain. After substitution of the asymptotic expansion for the slowly varying wavetrain into the governing equation and equating coefficients of the various powers of K to zero, the coefficient of the highest power of K yields an ordinary differential equation for the leading term of the asymptotic expansion as a function of the general phase coordinate. This equation is identical to that for the uniform periodic progressive wavetrain solution of the problem. Hence the dependence of the leading term of the asymptotic expansion for the slowly varying wavetrain on the general phase coordinate is the same as for the uniform wavetrain. Furthermore this leading term depends on four slowly varying parameters, among which the wavenumber and the frequency. For these parameters, determining the large-scale variations of the wavetrain, four equations are derived.

They satisfy the dispersion relation and also there is a relation between local wavenumber and local frequency expressing the conservation of wavecrests. The two remaining equations are obtained by imposing conditions of boundedness on the second term of the asymptotic expansion. This leads to a pair of integral relations involving the slowly varying parameters. It is shown that these relations also may be obtained by applying an appropriate averaging technique to some conservations laws of the problem.

The equations for the four slowly varying parameters are simplified by an asymptotic expansion with respect to the small amplitude/depth ratio. Only the lowest order non-linear effects are taken into account. After transformation in characteristic form a hyperbolic set of two equations involving wavenumber and amplitude uncouples. These equations are similar to the equations for the one-dimensional unsteady motion of a compressible gas with a fictitious adiabatic pressure-density relation. By transforming these equations into an axisymmetric wave equation it is possible to give an approximate solution to the initial value problem for slowly varying wavetrains.

In Chapter II one- and two-dimensional Boussinesq waves are dealt with and it is shown that also for these waves the "gas dynamics analogy" holds.

Chapter III is devoted to one-dimensional Stokes waves and the final results are identical to those obtained by Whitham's theory. In the Appendix a derivation is given of the uniform progressive periodic Stokes wave, developed in powers of the small amplitude/wavelength ratio.

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